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Stability of the Higgs mass in theories with extra dimensions†

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Abstract

We analyze the ultraviolet stability of the Higgs mass in recently proposed Kaluza-Klein models compactified on S_1/Z_2 or $S_1/(Z_2 \times Z_2')$, both at the field theory and string theory level. Fayet-Iliopoulos terms of $U(1)$ hypercharge are shown to be of vital importance for this discussion. Models with a single Higgs doublet seem to be generically affected by quadratic divergences.

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1 General remarks on Kaluza-Klein models

Perturbative calculations in quantum field theory are often accompanied by the presence of divergences. In the absence of anomalies, these divergences do not lead to inconsistencies, as their presence may be absorbed in the redefinition of the physical quantities of the model as for example the coupling constants of the theory. There are cases such as the Higgs sector in the Standard Model where these divergences are quadratic in the scale Λ or even quartic in the case of the vacuum energy. Therefore the physics of the Higgs sector is strongly dependent on the ultraviolet details of the theory. Indeed, in the Standard Model (SM), at one-loop level:

$$m_H^2 \propto \alpha\, \Lambda^2 \tag{1}$$

While stabilising the value of the Higgs mass to small values may be done (in a one loop order), this procedure must be repeated to every order in perturbation theory. This then brings a large amount of fine tuning in the theory.

A standard procedure to avoid this problem of the Standard Model is to introduce a supersymmetric extension of it, which is free of quadratic and quartic divergences, provided an additional condition (see later) for $U(1)_Y$ sector is respected. However, since supersymmetry is not an exact symmetry in Nature, a softly broken supersymmetry [1] at some scale m_{susy} is actually good enough for the purpose of alleviating the ultraviolet behaviour of the Higgs boson mass. In such a case the quadratic divergence in the Higgs sector is alleviated to one of (rather mild) logarithmic type. In this case the following relation emerges, with this solution to the UV sensitivity of the Higgs mass to hold beyond one loop order:

$$m_H^2 \propto m_{Susy}^2 \ln \frac{\Lambda}{m_{Susy}} \tag{2}$$

In the limit $m_{Susy} \to \infty$ the quadratic divergence is however restored, hence the need for a low energy (softly) broken supersymmetry. In this case the Higgs sector is only logarithmically sensitive to/dependent on the ultraviolet cut-off of the theory.

For supersymmetric model building purposes, condition (2) is however not enough for the absence of ultraviolet divergences of the Higgs mass beyond that of logarithmic type in (2) or that induced by the UV behaviour of the couplings of the theory. The reason for this is that there exists a contribution from the Fayet Iliopoulos (FI) term which brings in a quadratic dependence on Λ of the Higgs mass. This contribution is also proportional to the sum of the hypercharges of all massless complex scalars [2] of the model. Therefore the sum of these hypercharges must be zero to ensure the absence of quadratic divergences, and this is indeed the case of the Minimal Supersymmetric Standard Model (MSSM).

With growing interest in the physics of large extra dimensions various higher dimensional (supersymmetric) extensions of the standard model have been proposed. The simplest of these Kaluza-Klein models involve one additional compact dimension with radius R and employ either an orbifold compactification S_1/Z_2 or $S_1/(Z_2 \times Z_2')$ with mixed orbifold compactification and Scherk Schwarz breaking [3] of supersymmetry. One loop corrections to the Higgs mass have been computed in these models in ref.[4], [5], [6], [7], [8]. Employing a specific regularisation scheme one obtains a finite result:

$$m_H^2 \propto \frac{\alpha}{R^2} \tag{3}$$

with no explicit dependence on Λ. For particular models [6] it has been suggested that the finite result for the Higgs mass holds to all orders in perturbation theory [9]. This assertion is not necessarily justified as we will see later on.

The absence of any explicit cut-off dependence in the one loop expression (3) means that the models have one parameter less and this will allow numerical predictions for the Higgs mass. As with all effective models with additional (large) extra dimension(s), the new guise of the hierarchy problem is in this case that of dynamically "fixing" the value of the compactification scale, $1/R$ which in the aforementioned Kaluza-Klein models is required/implicitly assumed to be small, of TeV order. Otherwise, for $1/R$ large or of the order of the cut-off, one recovers the quadratic divergence of the Standard Model case, eq.(1), thus the importance of "fixing" R. One can also say that the need for small $1/R$ in Kaluza-Klein models is equally important to the need for a low scale of supersymmetry breaking in supersymmetric models, $m_{susy} \approx 1TeV$, see eqs.(2). This is because in both cases the Higgs (mass)^2 is proportional to the square of these quantities. A very large compactification scale $1/R$ would require a fine tuning of the couplings α to extremely small values, such that the prediction of the Higgs mass remain within the electroweak scale region, according to eq.(3). This can then significantly affect the phenomenological viability of the Kaluza-Klein models considered.

For a given model one can define the UV sensitivity of the Higgs mass (or amount of fine tuning to keep it close to electroweak scale)

$$\zeta_H \sim \frac{1}{m_H^2}\frac{dm_H^2}{d\ln\Lambda} \tag{4}$$

Other choices are possible, but the qualitative discussion below will not change. The UV sensitivity for Kaluza-Klein models, if (3) holds beyond one-loop, is controlled by the UV behaviour of the couplings α. The (gauge or Yukawa) couplings α of Kaluza-Klein models inevitably have some UV sensitivity to the cut-off due to their additional dimension(s). A one-loop corrected coupling in (3) amounts to a two-loop order correction to the Higgs mass. This one-loop coupling is in general expected to be linearly dependent on the scale in 5D Kaluza-Klein models. Therefore two loop Higgs mass is expected to have linear UV sensitivity $\zeta_H \sim d\ln\alpha/d\ln\Lambda \sim \alpha\Lambda R$, even though no apparent, explicit cut-off dependence is manifest at one-loop level, eq.(3). In a model by model approach it would be interesting to perform a relative comparison of this *linear* sensitivity induced beyond one loop, but suppressed by an additional power of the coupling to the logarithmic one of the MSSM case. This would be helpful in establishing accurately if an improvement from the MSSM case is possible beyond one loop order.

Generic five dimensional N=1 supersymmetric Kaluza-Klein models constructed recently fall in two classes: those whose low energy (massless) spectrum is that of the MSSM [5], [8] usually compactified on $M^4 \times S^1/Z_2$ and those compactified on $M^4 \times S^1/(Z_2 \times Z_2')$, where this is just the SM spectrum [6] Regardless of the details of the phenomenological viability of these models, one of their nice features is the presence of the electroweak symmetry breaking triggered by the boundary conditions one chooses for the 5D multiplets. Indeed the scalar potential in these models can be written in general as

$$\mathcal{V}(\phi) = \frac{1}{2}Tr\sum_{k=-\infty}^{\infty}\int\frac{d^4p}{(2\pi)^4}\ln\frac{p^2+m_{B_k}^2(\phi)}{p^2+m_{F_k}^2(\phi)} \tag{5}$$

where the masses of the Kaluza-Klein states has the structure

$$m_{B_k}^2 = \left[\frac{2}{R}\right]^2(k+q_B)^2 + M_\phi^2 \tag{6}$$

for bosonic states, with similar definition for the fermionic states. For the case of [6] $M_\phi = 0$ with the field dependence introduced via $q_{B(F)}$. At field theory level the apparent "freedom" of choosing the boundary conditions fixes the relative values of q_B and q_F. This in turns fixes the sign of the scalar potential (5) (and of its second derivative) and thus the presence or absence of the electroweak symmetry breaking by one loop radiative effects. In the more general context of string theory, compactified on $S_1/(Z_2 \times Z_2)$ with Scherk Schwarz breaking of supersymmetry, it turns out [13] that integer Kaluza-Klein momenta correspond to bosons $q_B = 0$ and half-integer momenta correspond to fermions, $q_F = 1/2$. This boundary condition would not lead to EW symmetry breaking by one loop effects.

The first class of models mentioned (see e.g. [8]) has some similarities with the MSSM, with the nice benefit of a one-loop cut-off independent Higgs mass, eq.(3) as compared to the MSSM case, eq.(2). They also have electroweak symmetry breaking triggered by the effects of Kaluza-Klein states. However, the couplings of the models run power-like with the scale. One can adopt two views of this situation. One is that the couplings (gauge or Yukawa) are just an input/fixed from experiment, and thus no UV sensitivity of m_H exists in one loop order or beyond, if the result (3) holds. The second view - and we share this point of view - is that in going beyond one loop order, the couplings' "running" is likely to re-introduce some UV sensitivity of the low energy physics[3] (m_H), as we discuss in the following. For a specific example [11] it was shown that Yukawa corrections to the Higgs mass induced by KK states bring in for m_H^2 a linear divergence at two loop level. This linear divergence can be re-absorbed in this case in the re-normalisation of Yukawa couplings, showing that a non-renormalisation of the superpotential still holds at one loop order (despite breaking supersymmetry by a Scherk Schwarz mechanism). Due to the non-renormalisable character of the models, it is not clear that all (beyond one loop) divergences may systematically be absorbed in the redefinition of physical quantities of the theory. Even so, the conclusion is that two-loop (Yukawa) corrected Higgs mass is a finite function of the re-scaled, linearly divergent, Yukawa coupling. Then some UV sensitivity of the Higgs mass will still exist, brought in by the coupling' dependence on the UV physics, even though no additional explicit cut-off dependence of m_H is present.

In a sense the discussion above suggests that Kaluza-Klein threshold corrections to the Higgs mass (or scalar potential) are related to the Kaluza-Klein threshold corrections to the gauge/Yukawa couplings of the model. In some cases the UV (cut-off or scale) dependence of m_H may then be redefined in terms of that of the couplings of the theory. For this reason it is sometimes said that, if the couplings of the theory are kept *fixed*, just as a final input, the Higgs mass is "calculable". This does not mean that m_H would be UV insensitive. Indeed, if (3) holds beyond one loop in a particular model, the UV sensitivity of m_H in that order (n) will still be (related to) that of couplings of the model in the corresponding order (usually $(n-1)$) of perturbation theory.

The second class of models mentioned is that with $S_1/(Z_2 \times Z_2')$ compactification. An explicit model is provided for example by the construction of ref.[6] with the massless spectrum chosen to be that of the SM. The model has some supersymmetric features, since it is obtained by compactifying N=1 supersymmetric theory in 5 dimensions on $M^4 \times S^1/(Z_2 \times Z_2')$, with KK states to fall into multiplets of this supersymmetry. However, the massless spectrum is that of the Standard Model with only one Higgs state. It is thus not clear to what extent the initial supersymmetry is able to protect the model from divergences, to give a (one loop) finite Higgs mass of type (3). In [12] concerns have been raised on the finiteness (beyond one loop) and on the UV insensitivity or amount of fine-tuning of the Higgs mass in such models, with further analysis on the link with string theory in [13]. In [14] it has been shown that the Higgs mass in this model has a quadratic UV divergence already at one loop order and eq.(3) does not apply. The divergence of the Higgs mass is a consequence of a divergent Fayet Iliopoulos (FI) term of $U(1)$ hypercharge. Since it turns out that FI terms are very important for the discussion of generic higher dimensional models [15], let us briefly describe in the next section the calculation of the FI term in generic models with $M_4 \times S_1/(Z_2 \times Z_2')$ compactification.

[3] This second point of view would correspond to the Wilsonian interpretation of the low energy effective field theory.

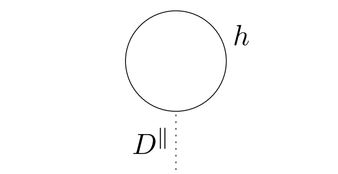
2 The Fayet-Iliopoulos term.

For the purpose of such evaluation [14] one can start with the 5D N=1 action describing a vector V and a hypermultiplet H [16] and compactify it on $M_4 \times S_1/(Z_2 \times Z_2')$. The vector multiplet has components $V = (A_M, \lambda, \Phi, D^a)$ with A_M the vector field, λ the Majorana gaugino, Φ a real scalar field, and D^a an $SU_R(2)$ iso-triplet of auxiliary scalars. The hypermultiplet $H = (h^i_\alpha, \zeta_\alpha, F^i_\alpha)$ contains a bi-doublet of $SU(2) \times SU_R(2)$ h^i_α (scalars), a hyperino ζ_α and auxiliary (bi-doublet) scalars F^i_a. The parity assignments for the Kaluza-Klein states are presented in the Table below

fields	$h^\alpha_{n\,\alpha}$	$h^\alpha_{n-\alpha}$	ζ^α_n	A^n_μ	A^n_5	λ^n_i	Φ_n	$D^{\|\|}_n$	$\vec{D}^\perp_n$
parities	$--$	$++$	$\alpha\,-\alpha$	$++$	$--$	$i\,-i$	$--$	$++$	$--$
modes n	≥ 1	≥ 0	≥ 0	≥ 0	≥ 1	≥ 0	≥ 1	≥ 0	≥ 1

with $\alpha, i = \pm$. Of the three auxiliary scalars, that form a triplet $\vec{D}$ under $SU_R(2)$ of the $N=1$ vector multiplet, two are odd under both parities $\vec{D}^\perp = (1 - \vec{a}_R \vec{a}_R^T)\vec{D}$, while the other one $D^{\|} = \vec{a}_R^T \vec{D}$ is even.

The corresponding diagram of the FI contribution to the self energy of a scalar is



The dotted line corresponds to the auxiliary field $D^{\|}$ of the Abelian gauge multiplet in 4D.

We investigate what happens to the FI term in the effective field theory coming from 5D with a mass spectrum of complex scalars of the hypermultiplet, in $S_1/(Z_2 \times Z_2')$ compactification. We take the charges of these scalars as $q_n^{++} = -q_n^{--} = 1$ because they belong to complex conjugate representations. The expression for the one loop contribution to the FI term whose diagram is presented above, is [14]

$$\xi = \sum_{n,\alpha} g\, q_n^{\alpha\alpha} \int \frac{d^4 p_4}{(2\pi)^4} \frac{1}{p_4^2 + (m_n^{\alpha\alpha})^2 + m^2} \tag{7}$$

where $m_n^{\alpha\alpha} = 2n/R$ and the sum for $\alpha = +$ is over $n \geq 0$, while for $\alpha = -$ over $n > 0$ in agreement with parity assignments of the above table.

The expression in (7) is considerably more complicated than the usual 4D case, each scalar mode of the infinite sum brings in a quadratic divergence, and a careful evaluation is required. To compute (7) one can transform the sum into an integral in the complex plane [17] as a regularisation procedure. With this regularisation we find an integral over the compact dimension of the following structure [14]

$$\xi = g \int \frac{d^{D_4} p_4}{(2\pi)^{D_4}} \int_\ominus \frac{d^{D_5} p_5}{2\pi i} \left[\frac{\mathcal{P}^{++}(p_5)}{p_4^2 + p_5^2 + m^2} - \frac{\mathcal{P}^{--}(p_5)}{p_4^2 + p_5^2 + m^2} \right]. \tag{8}$$

where we have introduced the "pole functions" [17] $\mathcal{P}^{++}$ and $\mathcal{P}^{--}$. For Im $p_5 > 0$, the pole functions may be written as [14]

$$\mathcal{P}^{\alpha\alpha} = \frac{1}{2} \left[-\frac{i}{2}\pi R + \frac{\alpha}{p_5} - \rho_-(p_5) \right], \qquad \rho_\alpha(p_5) = i\pi R \frac{e^{i\pi R p_5}}{1 + \alpha e^{i\pi R p_5}}. \tag{9}$$

For the case Im $p_5 < 0$ one finds [14]

$$\mathcal{P}^{\alpha\alpha} = \frac{1}{2}\left[\frac{i}{2}\pi R + \frac{\alpha}{p_5} + \rho_-(-p_5)\right], \tag{10}$$

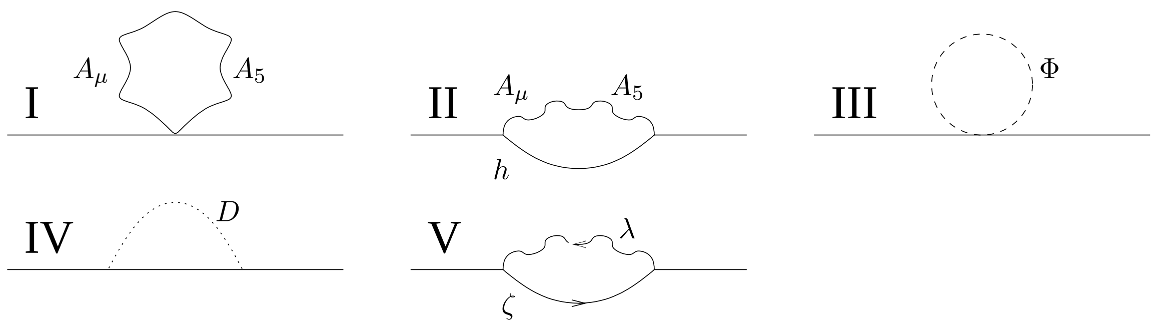
When integrated, the first term in $\mathcal{P}^{\alpha\alpha}$ accounts for genuine 5D divergences, the second (α/p_5) for 4D divergences while the remaining part gives the finite contributions.

Making use of the expressions of $\mathcal{P}^{\alpha\alpha}$ in the expression (8) we find that the result for ξ is:

$$\xi = g\,\frac{1}{2\pi i}\int\frac{d^{D_4}p_4}{(2\pi)^{D_4}}\int_{\ominus} d^{D_5}p_5\,\frac{1}{p_5}\frac{1}{p_4^2+p_5^2+m^2} = g\int\frac{d^{D_4}p_4}{(2\pi)^{D_4}}\frac{1}{p_4^2+m^2}. \tag{11}$$

We can safely take $D_5 = 1$ and remove the contour integration in (11) to obtain an expression similar to that in the 4D case. This just shows that the FI term at one loop is simply given by the contribution of the massless complex scalar ($m \to 0$) and leads to a quadratic divergence of the zero mode scalar mass (Higgs).

Further gauge corrections exist to the self energy of the zero mode scalar (identified as a Higgs state with continuous line in the diagrams below). They can be computed using the same procedure as for the FI term. The corresponding Feynman diagrams are:



A wavy line stands for a gauge field (A_μ, A_5), a wavy line with an arrow stands for gaugino λ, a line with an arrow - a hyperino ζ, a dashed line a real scalar Φ and a dotted line is an auxiliary field D^a. On the orbifold $S^1/(Z_2 \times Z_2')$ these fields are classified as even or odd under both parities.

The result of evaluating the above contributions to the zero mode scalar is

$$-i\Sigma_G = i\frac{7g^2}{16\pi^4}\left[\frac{1}{R/2}\right]^2\zeta(3) \tag{12}$$

Since there is no quadratic divergence in this result, one can safely conclude that it is not possible to cancel the quadratic divergence of the Fayet Iliopoulos term to the scalar mass.

The quadratic divergence of the correction to the Higgs massless mode eq.(11) holds for any finite R, since it is independent of the radius R of the compact dimension. Therefore, we conclude that it is also true in the limit $R \to \infty$. This signals that the boundary condition of the orbifolding procedure is not removed in limit $R \to \infty$. The de-compactification limit is thus not smoothly connected to a supersymmetric theory (obtained through compactification on a circle).

It is perhaps worth mentioning here that finite temperature arguments are sometimes used to justify a soft ultraviolet behaviour of the Higgs mass in 4D effective models with one additional compact dimension (5D N=1 supersymmetric model to start with). They proceed as follows. The inverse radius of the compact dimension corresponds to the temperature. Supersymmetry in the initial 5D model then ensures a vanishing of the potential/Higgs mass in the $R \to \infty$ (or $T = 0$) limit. Finite values of R would then trigger only finite corrections to the scalar potential/Higgs mass, by analogy with finite temperature calculations of the free energy, since in going from $T = 0$ to $T \neq 0$ only finite corrections could appear, if the limit $T \to 0$ ($R \to \infty$) is smooth. However it seems to be impossible to reconcile this mechanism with the phenomenological requirement to obtain a chiral spectrum in 4D, while smoothly varying the radius from $R \to \infty$ of the N=1 supersymmetric limit in 5D to a broken supersymmetry phase. An orbifold-like compactification is necessary, but then the limit $R \to \infty$ on the orbifold does not restore the full initial supersymmetry at the fixed points, as we just discussed in the example above. In other words finite temperature arguments may apply to a circle compactification of the extra dimension (non-chiral models), but not necessarily to a manifold with fixed points (orbifold compactifications).

At an early stage of the discussion, a further explanation for the finiteness of the Higgs mass was the argument that a counterterm for the Higgs mass was forbidden by the symmetries of the theory. Various symmetries have been suggested in that direction. The result of ref. [14], however, suggests a new counterterm in the theory that is closely connected to the Higgs mass. The analysis of (localized) anomalies at the fixed points [18], [19], [20], [21] clarifies the nature of exact symmetries of the model, which in fact allow a counterterm for the Higgs mass in the $S_1/(Z_2 \times Z_2')$ model [21].

The conclusion we draw from the above calculation is that a generic 5D N=1 supersymmetric model, with a compactification $S_1/(Z_2 \times Z_2')$ with massless sector identical to that of the SM will have a quadratic divergence for the Higgs mass at one-loop level[4]. In such case [6] an improvement from the SM UV behaviour of eq.(1), is not possible. A suggestion to avoid such problem may be to introduce an additional Higgs state as zero mode, at the price of bringing an extra parameter with implications on the predictive power of the model.

3 Kaluza-Klein models - the view from string theory

What is the situation in string theory? Afterall this is supposed to provide a more consistent framework for investigating corrections to the Higgs mass and scalar potential. At string level, the latter has a correspondent in the vacuum energy, which receives corrections from the string Kaluza-Klein and winding modes as well as massive states.

Comparing the scalar potential of Kaluza-Klein models with the results from string theory is not straightforward. The models must have similar spectrum, orbifold compactification and supersymmetry breaking mechanism. The Kaluza-Klein models must also be free of anomalies not only in 4 dimensions, but also in their 5D embedding. This is important for it may happen that anomalies in 5D, localised at the fixed points be present in some Kaluza-Klein models considered. In string theory their absence is ensured by some variation of Green-Schwarz mechanism or by the twisted states, whose existence/absence in phenomenological studies of Kaluza-Klein models has so far been somewhat arbitrary. It has been shown [18], (see also [19]) in a field theory approach that this is a non-trivial issue and that for some models anomalies localised at the fixed points may be present in 5D, with implications for the overall consistency of the models. We thus assume that these difficulties are overcome, and one is able to identify a consistent Kaluza-Klein model which is indeed the low energy limit of a heterotic or type I string model.

In the string context the phenomenological requirement that R be of particular size requires a dynamical mechanism to fix the point in moduli space giving this value. Further, the couplings receive

[4]This seems in disagreement with findings in [10].

some string scale dependence/sensitivity, thus the need for including the (string) threshold corrections to the gauge or Yukawa couplings for the model considered. Such threshold corrections are due to Kaluza-Klein and winding states associated with the extra compact dimension(s) considered. Our previous discussion at the level of Kaluza-Klein models on the link between UV sensitivity of the couplings and of the Higgs mass/scalar potential has then a correspondent at the string level. The (string) threshold corrections to the gauge/Yukawa couplings may be related to those to the vacuum energy [13]. Therefore the UV sensitivity of the Higgs mass/vacuum energy and of the gauge couplings are related at string level as well.

Since most of the results of 5D N=1 supersymmetric Kaluza-Klein models of growing phenomenological interest invoked a Scherk Schwarz mechanism for supersymmetry breaking, we consider such a case at the level of (heterotic) string theory. The case of type I string is only briefly addressed. We present the results [13] for the vacuum energy/cosmological constant for a $S_1/(Z_2 \times Z_2')$ orbifold compactification, although the results are rather generic to other compactifications. We limit ourselves to listing some comparative features of the string results, to stress the "regularisation" role of the winding modes and modular invariance.

Consider therefore a heterotic orbifold model with N=1 supersymmetry in four dimensions, which has untwisted N=4 and N=2 sub-sectors. Supersymmetry is broken in these sub-sectors of the orbifold by a Scherk-Schwarz mechanism, by the following choice of the shifts of the internal charges [22]:

$$n \to n, \qquad m \to m + p - \frac{1}{2} n, \qquad p \to p - n \tag{13}$$

where n, m, p are respectively the winding and momentum numbers with respect to the compactified direction, while p is the internal $U(1)$ fermion charge. A Scherk-Schwarz mechanism with respect to the untwisted planes of the orbifold will break N=2 and N=4 supersymmetry of the completely untwisted sector to N=0, while the N=1 sub-sector is left untouched (no mass shifts in this sub-sector).

Detailed calculations of the vacuum energy in string models with such a mechanism for supersymmetry breaking give the result of eq.(14) below, which unlike that of Kaluza-Klein models, is well defined (finite) in the ultraviolet region. The only model dependence of the result is manifest in the coefficients γ_N. It is important to remark at this stage that the limit of integration in eq.(14) from deep ultraviolet $t \geq 0$ is introduced by the presence of winding states. Essentially this comes about in the following way. Initially the vacuum energy is defined as equal to two sums over winding and momentum modes of an integral over the fundamental domain $\mathcal{F}_2$ of the string. A technical argument [23] enables one to re-define the summation indices into linear combinations of winding and momentum modes. One of these re-defined sums when applied to the fundamental domain of integration "unfolds" it to the half strip integration region defined by[5] $\mathcal{H} = \{\tau| -1/2 \leq \tau_1 \leq 1/2; 0 \leq \tau_2 < \infty\}$. In this "unfolding" procedure the winding modes' contribution is essential, and they ensure that eq.(14) is well defined in the deep ultraviolet. The second (remaining) sum is over the integer p which is a mixture of (Poisson re-summed) Kaluza-Klein and of winding numbers and it is present in the formula below:

$$V_h(R) = R \int_0^\infty \frac{d\,\tau_2}{\tau_2^{7/2}} \sum_{p>0} \sum_{N \geq 0} \{1 - (-1)^p\}\, e^{-\frac{\pi (R/2)^2}{\tau_2} p^2} e^{-4\pi\tau_2 N} \gamma_N \tag{14}$$

This gives the full (heterotic) string result for the vacuum energy, generic to orbifold compactifications with Scherk Schwarz breaking of supersymmetry in the untwisted sector. The result is finite and no

[5] Further, the integral over τ_1 essentially projects all massive states to keep only those of equal mass levels (physical) and one is left with the integral over τ_2 only, eq.(14).

regularisation is required for the UV region $\tau_2 \to 0$. In this region most contributions are exponentially suppressed in (14), except those of small radius (string units) which may still contribute. The latter may be interpreted as corresponding to Kaluza-Klein states of mass larger than the string scale, $k/R > M_{string}$. The presence of such states in the string framework is justified, unlike the case of the (effective) field theory where they may also be manifest, with a less clear physical meaning [12].

After some algebra the (heterotic) string result can be written as (using $x_N = exp(-4\pi R N^{1/2})$ and $\mathcal{L}i_n(x) = \sum_{k\geq 1} x^k/k^n$)

$$\begin{aligned} V_h(R) \;\; = \;\; & \frac{93\zeta(5)\gamma_0}{64\pi^2 R^4} + \frac{4}{R^2}\sum_{N>0} \gamma_N \; N \left[\mathcal{L}i_3(x_N) + \frac{3}{4\pi R\sqrt{N}}\; \mathcal{L}i_4(x_N) + \frac{3}{16\pi^2 R^2 N}\; \mathcal{L}i_5(x_N) \right. \\ & \left. -\mathcal{L}i_3(-x_N) - \frac{3}{4\pi R\sqrt{N}}\; \mathcal{L}i_4(-x_N) - \frac{3}{16\pi^2 R^2 N}\; \mathcal{L}i_5(-x_N) \right] \; , \end{aligned} \tag{15}$$

with the leading term proportional to γ_0/R^4 and remaining terms exponentially suppressed. γ_0 vanishes if the ground state is supersymmetric. The model dependent coefficients γ_N have contributions from $N{=}4$ and $N{=}2$ sub-sectors of the initial orbifold, broken to $N = 0$ by Scherk Schwarz mechanism.

An intriguing feature of the heterotic result eq.(14) is that it is close to the type I result for vacuum energy, with Scherk Schwarz supersymmetry breaking. The type I result is [13] (for full references and review see [24])

$$V_I(R) = \int_0^\infty \frac{dt}{t^3} \sum_{n\in Z} (-1)^n e^{-\pi t n^2/R^2} \sum_{N\geq 0} \tilde{\gamma}_N e^{-\pi t N} \Big|_{reg.} \tag{16}$$

Unlike the heterotic string case, a regularisation is necessary in type I string case. In (16) “reg” stands for a regularisation of the divergence introduced by the presence of $n = 0$ (for $N = 0$) Kaluza-Klein state. The need for a regularisation in type I case is not new, it is manifest in other calculations as well (for example threshold corrections to the gauge couplings [25]). This is due to the absence of modular invariance in these models which at heterotic level ensures a finite result. The absence of winding modes requires a regularisation procedure, and a similarity of type I results to those in Kaluza-Klein models with momentum modes only (no windings), will also exist. It is considered that the state $n = 0$ is not present if eq.(16) is regularised in the transverse closed string channel and one-particle irreducible diagrams are subtracted [26]. Additional arguments for such a regularisation and the absence of the divergence when summing over all open string diagrams, are represented by the (requirement of) tadpoles cancellation [25].

For comparison of the (heterotic) string result with that of Kaluza-Klein models, we note that the former, eq.(14) can be re-written after introducing a $p = 0$ state and then Poisson re-summing over p

$$V_h(R) = \int_{\epsilon^2\alpha'}^\infty \frac{d\,z}{z^3} \sum_{s=-\infty}^{\infty} \left\{ e^{-\frac{\pi z}{(R/2)^2} s^2} - e^{-\frac{\pi z}{(R/2)^2}(s+1/2)^2} \right\} \sum_{N\geq 0} e^{-\pi z N} \gamma_N \; . \tag{17}$$

This expression is very similar to that of *regularised* Kaluza-Klein models (except the summation over massive modes N) and that of type I case, eq.(16). The presence of the lower limit of integration $\epsilon^2\alpha'$ in (17) is needed for bosonic and fermionic terms under the integral be each well defined in the UV (for $N = 0$, $z \to 0$) [13]. Therefore a regularisation is required for a field theory interpretation/limit. Strictly speaking, the summation index s in (17) is a “mixture” of Kaluza-Klein number and (Poisson re-summed) winding number, which gives for the contribution to V of the momentum modes alone a further constraint $|s| \leq (M_S R)/\epsilon$. This arises because the cut-off in (17) $\epsilon^2\alpha'$ excludes the deep ultraviolet ($\tau_2 \to 0$) momentum region, to retain under the integral over z only the momentum modes with mass range $1/z < 1/(\epsilon^2\alpha')$. In the field theory case with an infinite string scale ($\alpha' = 0$), this constraint is removed. One assumes that *infinitely* many winding modes (and their effects) are entirely decoupled. Then from (13) at zero windings $n = 0$ a "discrete shift symmetry" emerges for Kaluza-Klein level $k \to k + p$, to require one sum over the whole Kaluza-Klein tower. A result with structure close to that of the full heterotic string result (14), (see also (15), (17)) - which includes winding modes' effects - is then obtained. In such case, the field theory results and heterotic/type I string results are similar if: in eqs.(15), (17) one makes the formal replacement $N \to M_\phi^2$ and ignores the sum over the massive modes N. In this case the mass of KK states of field theory eq.(6) will appear in the exponents in the integral (17) and in (15).

Since a field theory regime corresponds to a string calculation with an infinite string scale, the string and field theory results may in general be matched exactly only in this limit. Additional finite string effects which vanish in this limit [27] may not be recovered by the field theory calculation. In general, in the absence of a full string calculation to compare with, one cannot state, on field theory grounds, that the *full* UV behaviour of a model with extra-dimensions is that found by using momentum modes only. Winding modes, although massive in the field theory limit $\alpha' \to 0$, can have additional effects. Their presence enables the symmetries of the string (modular invariance) which require integrations take place over the fundamental domain, ensuring an UV finite (heterotic string) result. They then play a (regularisation) role in the UV behaviour of the string models in the limit $\alpha' \to 0$. In some cases, for gauge couplings [27], they also control the exact value of the coefficient of the divergence the string result has in $\alpha' \to 0$ limit, which is just the limit of the field theory regime. To conclude, to find the *full* UV behaviour of a model we argued that from a string theory perspective it is more appropriate to consider the limit $\alpha' \to 0$ on the full string result. This should unambiguously determine the UV limit of the model, without being subject to a regularisation scheme dependence induced by field theory calculations.

4 Conclusions

We have seen that the request for a better UV behaviour (than in the SM, eq.(1)) of the Higgs mass, in models with extra dimensions, leads to serious constraints on the spectrum and the interactions of the theory. This should not be too surprising as the higher dimensional theory is non-renormalisable. We discussed the ultraviolet sensitivity of Higgs mass in Kaluza-Klein models compactified on S_1/Z_2 and $S_1/(Z_2 \times Z_2')$. For the latter class of models with massless modes as in the SM with one Higgs state, a FI term is generated, which introduces a quadratic divergence for the mass of the Higgs particle. In the more general context of a string theory calculation for the vacuum energy, the "regularisation role" of the winding modes was emphasized, as this ensures the finiteness of the (heterotic string) result. This is unlike the type I string case and Kaluza-Klein effective models, which do not have winding contributions to the vacuum energy/scalar potential and where a regularisation is required. To find the right UV behaviour of a model we argued it is more appropriate to consider the limit $\alpha' \to 0$ of the full string result, which should unambiguously determine the full UV behaviour of the model.

The requirement of mild UV sensitivity of the Higgs mass leads to significant constraints on the possible higher dimensional extensions of the $SU(3) \times SU(2) \times U(1)$ standard model. FI terms of U(1) hypercharge seem to play a central role in this discussion, most notably in the (non-supersymmetric) $S_1/(Z_2 \times Z_2')$ case. Even in the $N = 1$ supersymmetric models, as e.g. S_1/Z_2, divergent FI-terms localized at the orbifold fixed points [15] could lead to a destabilization of the theory. Generically, these problems can be solved by a sufficient amount of supersymmetry and the presence of a second

Higgs multiplet with opposite hypercharge. This then leads us back to models very similar to the MSSM with softly broken supersymmetry.

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